Linear Programming

ISC Class 12 Previous Years Board Questions

With Answers

2008 to Sem2-2022

1.	A manufacturer produces two types of steel trunks. He has two machines, A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type requires 3 hours on machine A and 2 hours on machine B. Machine A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of ₹ 30 per trunk on the first type of trunk and ₹ 25 per trunk on the second type. Formulate a linear programming problem to find out how many trunks of each type he must make each day to maximize his profit.	(ISC 2008)
2.	A new cereal, formed of a mixture of bran and rice, containing at least 88 grams of protein and at least 36 milligrams of iron per kilogram . Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilogram and that rice contains 100 grams of protein and 30 milligrams of iron per kilogram, find the minimum cost of producing a kilogram of this new cereal if bran costs ₹ 28 per kilogram and rice costs ₹25 per kilogram.	(ISC 2009)
3.	A company produces two types of items, P and Q. Manufacturing of both items requires the metals gold and copper. Each unit of item P requires 3 gms of gold and 1 gm of copper while that of item Q requires 1 gm of gold and 2 gms of copper. The company has 9 gms of gold and 8 gms of copper in its store. If each unit of item P makes a profit of ₹ 50 and each unit of item Q makes a profit of ₹ 60, determine the number of units of each item that the company should produce to maximize profit. What is the maximum profit?	(ISC 2010)

4.	A manufacturer manufactures two types of tea-cups A and B. Three machines are needed for manufacturing the teacups. The time in minutes required for manufacturing each cup on the machines is given below. Each machine is available for a maximum of six hours per day. If the profit of each cup of type A is ₹ 1·50 and that on each cup of type B is ₹ 1·00, find the number of cups of each type that should be manufactured in a day to get maximum profit.	(ISC 2011)
5.	Two tailors P and Q earn ₹ 150 and ₹ 200 per day respectively. P can stitch 6 shirts and 4 trousers a day, while Q can stitch 10 shirts and 4 trousers per day. How many days should each work to produce at least 60 shirts and 32 trousers at minimum labour cost?	(ISC 2012)
6.	A mill owner buys two types of machines A and B for his mill. Machine A occupies 1000 sqm of area and requires 12 men to operate it; while machine B occupies 1200 sqm of area and requires 8 men to operate it. The owner has 7600 sqm of area available and 72 men to operate the machines. If machine A produces 50 units and machine B produces 40 units daily, how many machines of each type should he buy to maximize the daily output? Use linear programming to find the solution.	(ISC 2013)
7.	A company manufactures two types of toys A and B. A toy of type **A** requires 5 minutes of cutting and 10 minutes for assembling. A toy of type **B** requires 8 minutes for cutting and 8 minutes for assembling. There are 3 hours available for cutting and 4 hours are available for assembling the toys in a day. The profit is ₹ 50 each on a toy of type A and ₹ 60 each on a toy of type **B**. How many toys of each type should the company manufacture in a day to maximize the profit? Use linear programming to find the solution.	(ISC 2014)

Type of Cup	Times in Minutes		
	Machine I	Machine II	Machine III
A	12	18	6
B	6	0	9

8.	A dietician wishes to mix two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food are given below: One kg of food X costs ₹ 24 and one kg of food Y costs ₹ 36. Using linear programming, find the least cost of the total mixture which will contain the required vitamins.	(ISC 2015)
9.	A company manufactures two types of products A and B. Each unit of A requires 3 grams of nickel and 1 gram of chromium, while each unit of B requires 1 gram of nickel and 2 grams of chromium. The firm can produce 9 grams of nickel and 8 grams of chromium. The profit is ₹ 40 on each unit of product of type A and ₹ 50 on each unit of type B. How many units of each type should the company manufacture so as to earn maximum profit? Use linear programming to find the solution.	(ISC 2016)
10.	A farmer has a supply of chemical fertilizer of type A which contains 10% nitrogen and 6 % phosphoric acid and of type B which contains 5 % nitrogen and 10% phosphoric acid. After soil test, it is found that at least 7 kg of nitrogen and same quantity of phosphoric acid is required for a good crop. The fertilizer of type A costs ₹ 5.00 per kg and the type B costs ₹ 8.00 per kg. Using linear programming, find how many kilograms of each type of the fertilizer should be bought to meet the requirement and for the cost to be minimum. Find the feasible region in the graph.	(ISC 2017)
11.	A manufacturing company makes two types of teaching aids **A** and **B** of mathematics for class X. Each type of A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹ 80 on each piece of type A **and** ₹ 120 on each piece of type **B**. How many pieces of type **A** and type **B** should be manufactured per week to get a maximum profit? Formulate this as Linear programming problem and solve it. Identify the feasible region from the rough sketch.	(ISC 2018)

Food	Vitamin A	Vitamin B	Vitamin C
X	1 unit	2 units	3 units
Y	2 units	2units	1 unit

<table>
<tr><td>12.</td><td>A carpenter has 90, 80 and 50 running feet respectively of teak wood, plywood and rosewood which is used to produce product A and product B. Each unit of product A requires 2, 1 and 1 running feet and each unit of product B requires 1, 2 and 1 running feet of teak wood, plywood and rosewood respectively. If product A is sold for ₹ 48 per unit and product B is sold for ₹ 40 per unit, how many units of product A and product B should be produced and sold by the carpenter, in order to obtain the maximum gross income?
Formulate the above as a linear Programming Problem and solve it, indicating clearly the feasible region in the graph.</td><td>(ISC 2019)</td></tr>
<tr><td>13.</td><td>A company uses three machines to manufacture two types of shirts, half sleeves and full sleeves. The number of hours required per week on machine M_1, M_2 and M_3 for one shirt of each type is given in the following table:
<table><tr><td></td><td>M_1</td><td>M_2</td><td>M_3</td></tr><tr><td>Half sleeves</td><td>1</td><td>2</td><td>8/5</td></tr><tr><td>Full sleeves</td><td>2</td><td>1</td><td>8/5</td></tr></table>None of the machines can be in operation for more than 40 hours per week. The profit on each half sleeve shirt is ₹ 1 and the profit on each full sleeve shirt is ₹ 1·50. How many of each type of shirts should be made per week to maximize the company's profit?</td><td>(ISC 2020)</td></tr>
<tr><td>14.</td><td>A Manufacturer has two machines X and Y that may run at the most 360 minutes in a day to produce two types of toys A and B.

To produce each Toy A, machines X and Y need to run at the most 12 minutes and 6 minutes respectively.

To produce each toy B, machines X and Y need to run at the most 6 minutes and 9 minutes respectively.

By selling the toys A and B, the manufacturer makes the profits of ₹30 and ₹20 respectively.

Formulate a Linear Programming Problem and find the number of toys A and B that should be manufactured in a day to get maximum profit.</td><td>(ISC Sem2-2022)</td></tr>
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Answer

1.	$Maximize\ z = 30\,x + 25\,y\ subject\ to\ the\ constraints$ $3x + 3y \leq 18, 3x + 2y \leq 15, x \geq 0, y \geq 0$ Maximum profit is attained when 3 trunks of 1st type and 3 trunks of 2nd type is made per day. Maximum profit is ₹ 165.	(ISC 2008)
2.	$Minimize\ z = 28\,x + 25\,y\ subject\ to\ the\ constraints$ $80\,x + 100\,y \geq 88,\ 40x + 30y \geq 36,\ x \geq 0, y \geq 0.$ Cost is minimum when 0.6 kg bran and 0.4 kg of iron is used. Minimum cost of producing cereal per kg is ₹ 26.80.	(ISC 2009)
3.	$Maximize\ z = 50x + 60y\ subject\ to\ the\ constraints$ $3x + y \leq 9, x + 2y \leq 8,\quad x \geq 0, y \geq 0$ For maximum profit, the company should produce 2 units of item P and 3 units of item Q. Maximum profit is ₹ 280.	(ISC 2010)
4.	$Maximize\ z = 1.50\,x + 1.00\,y\ subject\ to\ the\ constraints$ $2x + y \leq 60,\ x \leq 20,\ 2x + 3y \leq 120,\ x \geq 0,\ y \geq 0.$ 15 cups of type A and 30 cups of type B should be manufactured per day to get maximum profit. Maximum profit is ₹ 52.5.	(ISC 2011)
5.	$Minimize\ z = 150\,x + 200y\ subject\ to\ the\ constraints$ $6x + 10y \geq 60,\ 4x + 4y \geq 32,\quad x \geq 0,\ y \geq 0.$ To minimize labour cost, P will work for 5 days and Q will work for 3 days. Minimum labour cost is ₹1350.	(ISC 2012)
6.	$Maximize\ z = 50x + 40\,y\ subject\ to\ the\ constraints$ $5x + 6y \leq 38, 3x + 2y \leq 18, x \geq 0, y \geq 0$ He should buy 4 machines of type A and 3 machines of type B to maximize daily output. Maximum output 320 units daily.	(ISC 2013)

7.	$Maximize\ z = 50x + 60y\ subject\ to$ $5x + 8y \leq 180, \quad 10x + 8y \leq 240, \quad x \geq 0, \quad y \geq 0.$ Maximum profit is ₹ 1500 when 12 units of type **A** and 15 units of type **B** is produced.	(ISC 2014)
8.	$Minimize\ z = 24x + 36\ y\ subjects\ to\ the\ constraints$ $x + 2y \geq 10,\ x + y \geq 6,\ 3x + y \geq 8,\ x \geq 0,\ y \geq 0.$ Least cost is ₹ 192 and is obtained when 2 kg of food X is mixed with 4 kg of food Y.	(ISC 2015)
9.	$Maximize\ z = 40x + 50y\ subject\ to\ the\ constraints$ $3x + y \leq 9, \quad x + 2y \leq 8, \quad x \geq 0, y \geq 0$ Maximum profit is ₹ 230, when 2 units of type A and 3 units of type B are produced.	(ISC 2016)
10.	$Minimize\ z = 5x + 8y\ subject\ to\ the\ constraints$ $2x + y \geq 140,\ 3x + 5y \geq 350 \quad x \geq 0,\ y \geq 0$ Minimum cost is ₹ 570 and is obtained when 50 kg of fertilizer A is mixed with 40 kg of fertilizer B.	(ISC 2017)
11.	$Maximize\ z = 80x + 120y\ subject\ to\ the\ constraints$ $3x + 4y \leq 60,\ x + 3y \leq 30,\ x \geq 0, y \geq 0$ Maximum profit is ₹ 1680 and is obtained when 12 units of type A and 6 units of type B is produced per week.	(ISC 2018)
12.	$Maximize\ z = 48x + 40y\ subject\ to\ the\ constraints:$ $2x + y \leq 90,\ x + 2y \leq 80,\ x + y \leq 50,\ x \geq 0,\ y \geq 0$ Maximum gross income is ₹ 2,320. For maximum gross income the carpenter should make 40 units of product A and 10 units of product B.	(ISC 2019)
13.	Maximize $z = x + 1{\cdot}50\ y$ subject to the constraints: $x + 2y \leq 40,\ 2x + y \leq 40,\ x + y \leq 25,\ x \geq 0,\ y \geq 0\ .$ Maximum profit is ₹ $32{\cdot}5$ and 10 half sleeve shirts and 15 full sleeves shirts should be made per week to maximize profit.	(ISC 2020)
14.	Maximize Profit (in ₹), $z = 30\ x + 20\ y$ Subject to the constraints $12x + 6y \leq 360$ or $2x + y \leq 60$ $6x + 9y \leq 360$ or $2x + 3y \leq 120$ $x \geq 0, \quad y \geq 0$ When 15 toys of type A and 30 toys of type B are manufactured per day, Profit is maximum and maximum profit is ₹ 1050.	(ISC 2022)

Solutions of

Linear Programming

ISC Class 12 Previous Years Board Questions

2008 to 2020

1. A manufacturer produces two types of steel trunks. He has two machines A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine on B. The second type requires 3 hours on machine A and 2 hours on machine B. Machine A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of ₹ 30 per trunk on the first type of trunk and ₹ 25 per trunk on the second type. Formulate a linear programming problem to find out how many trunks of each type he must make each day to maximize his profit. (ISC 2008)

Solution : Let number of first type of trunks made per day $= x$

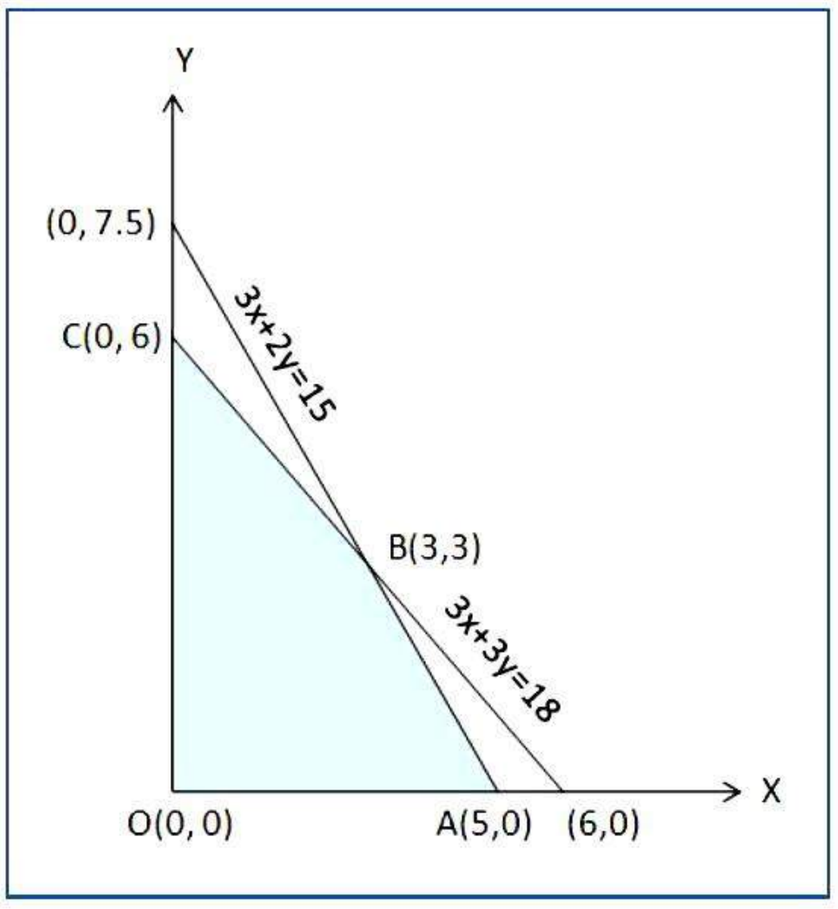
Number of second type of trunk made per day $= y$

The mathematical formulation of the given LPP is

Maximize profit (in₹) $P = 30x + 25\,y$

Subject to the constraints

$$3x + 3y \leq 18$$
$$3x + 2y \leq 15$$
$$x \geq 0,\ y \geq 0$$



The shaded region OABC is feasible region.

The corner points are O(0,0), A(5,0) , B(3,3)and C(0,6) .

at O(0,0) , P= 0

at A(5,0) , P = 30 x 5 + 25 x 0= 150

at B(3,3) , P = 30 x3+25 x 3 = 165

at C(0,6) , P = 25 x 6 = 150

So maximum profit is ₹ 165

and it is obtained at B(3,3)

So maximum profit is attained when 3 trunks of 1st type and 3 trunks of 2nd type is made per day.

2. A new cereal formed of a mixture of bran and rice, containing at least 88 grams of protein and at least 36 milligrams of iron per kilogram. Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilogram and that rice contains 100 grams of protein and 30 milligrams of iron

per kilogram, find the minimum cost of producing a kilogram of this new cereal if bran costs ₹ 28 per kilogram and rice costs ₹ 25 per kilogram. (ISC 2009)

Solution : The given data may be put in the following tabular form :

		Protein (g/kg)	Iron (mg/kg)	Cost (₹ /kg)
Bran	x	80	40	28
Rice	y	100	30	25
Requirement		88	36	

Let x kg of bran and y kg of rice be mixed to form the cereal.

Then the problem can be formulated as an L.P.P. as follows :

Minimize total cost (in ₹) $z = 28x + 25y$

subject to the constraints

$$80x + 100y \geq 88 \quad i.e. \quad 20x + 25y \geq 22$$
$$40x + 30y \geq 36 \quad i.e. \quad 20x + 15y \geq 18$$
$$x \geq 0, \quad y \geq 0$$

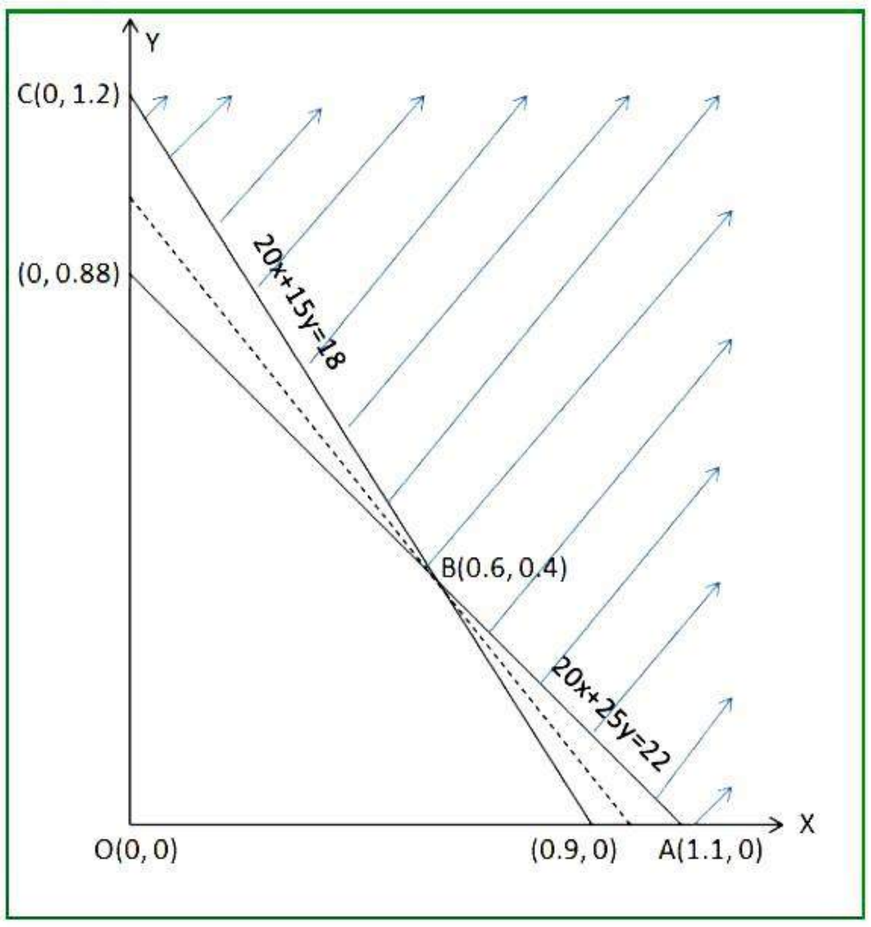
The shaded unbounded region is the feasible region.

The corner points are

$A(1{\cdot}1,0)$, $B(0{\cdot}6, 0{\cdot}4)$, $C(0, 1{\cdot}2)$

At the corner points ,

the value in ₹ of $z = 28x + 25y$ are :



at $A(1{\cdot}1, 0)$, $z = 28 \times 1{\cdot}1 + 0 = 30{\cdot}8$,

at $B(0{\cdot}6, 0{\cdot}4)$, $z = 28 \times 0{\cdot}6 + 25 \times 0{\cdot}4 = 26{\cdot}8$

at $C(0, 1{\cdot}2)$, $z = 0 + 25 \times 1{\cdot}2 = 30$

$\therefore$ minimum value of $z =$ ₹ $26{\cdot}8$

and it occurs at the point $B(0{\cdot}6, 0{\cdot}4)$.

Let us draw the line $28x + 25y = 26{\cdot}8$ (shown dotted). The half plane $28x + 25y < 26{\cdot}8$ has no common points with the feasible region. So, z has minimum value. Minimum value of z is ₹ $26{\cdot}8$.

Hence when $0{\cdot}6$ kg of bran and $0{\cdot}4$ kg of rice is mixed, so that the weight of mixture is

$0{\cdot}6$kg + $0{\cdot}4$ kg = 1 kg, the minimum cost of 1kg of mixture is ₹ $26{\cdot}8$.

3. A company produces two types of items P and Q. Manufacturing of both items requires the metals gold and copper. Each unit of item P requires 3 gms of gold and 1 gm of copper while that of item Q requires 1 gm of gold and 2 gms of copper. The company has 9 gms of gold and 8 gms of copper in its store. If each unit of item P makes a profit of ₹ 50 and each unit of item Q makes a profit of

₹ 60, determine the number of units of each item that the company should produce to maximize profit. What is the maximum profit? (ISC 2010)

Solution : Let the number of units of item A to be produced be x and that of B to be produced be y. Let z be the total profit in rupees.

The given data can be put in tabular form as :

Items		gold	copper	Profit per unit (₹)
P	x	3	1	50
Q	y	1	2	60
availability		9	8	

Thus, mathematical formulation of given LPP is

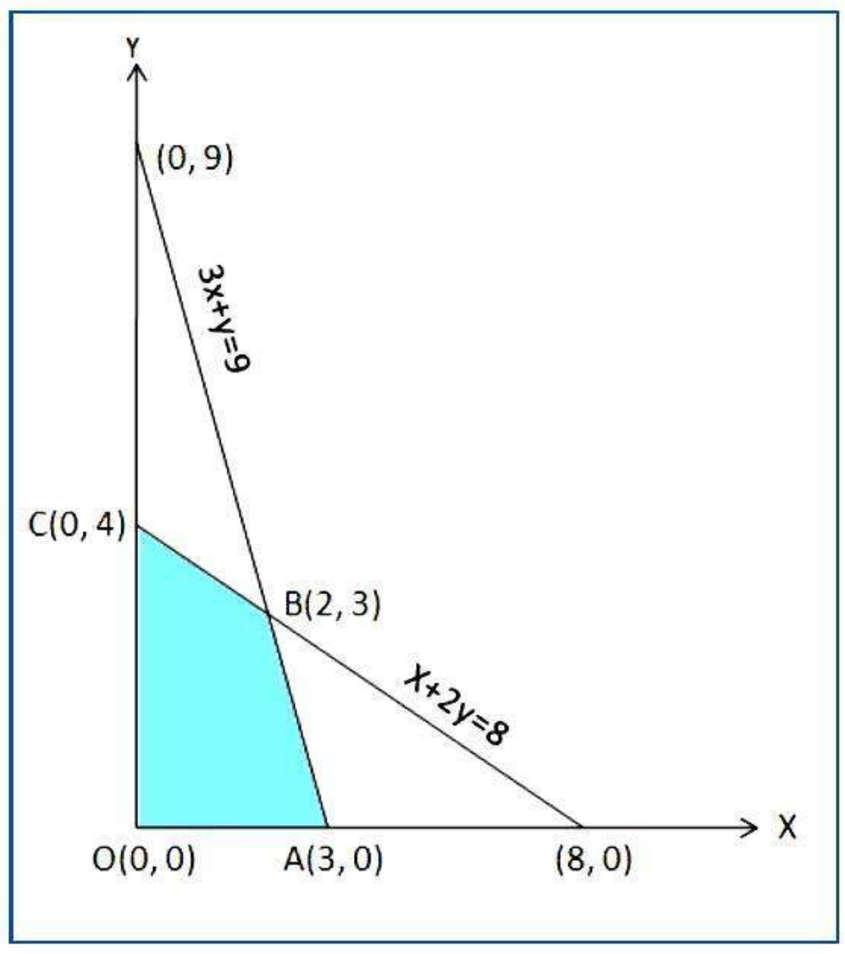
Maximize (in₹) $z = 50x + 60\,y$

Subject to the constraints $3x + y \leq 9$

$x + 2y \leq 8$

$x, y \geq 0$

The shaded region OABC is the feasible region.



The corner points are :

$O(0,0),\ A(3,0),\ B(2,3),\ C(0,4).$

at $O(0,0), z = 0$.

at $A(3,0), z = 50 \times 3 + 60 \times 0 = 150.$

at $B(2,3),\ z = 50 \times 2 + 60 \times 3 = 280.$

at $C(0,4), z = 50 \times 0 + 60 \times 4 = 240.$

$\therefore$ z is maximum at $x = 2$ and $y = 3$.

$\therefore$ for maximum profit , the company should produce 2 units of item P and 3 units of item Q.

$\therefore$ maximum profit is ₹ 280.

4. A manufacturer manufactures two types of tea-cups, A and B. Three machines are needed for manufacturing the teacups. The time in minutes required for manufacturing each cup on the machines is given below:

Type of cup	Times in minutes		
	Machine I	Machine II	Machine III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of six hours per day. If the profit on each cup of type A is ₹ 1.50, and that on each cup of type B is ₹ 1.00, find the number of cups of each type that should be manufactured in a day to get maximum profit. (ISC 2011)

Solution : Let x and y be the number of tea-cups of types A and B respectively.

The problem can be formulated as an L.P.P. as follows:

Maximize the profit (in ₹) $z = 1{\cdot}5x + 1\,y$

subject to the constraints ,

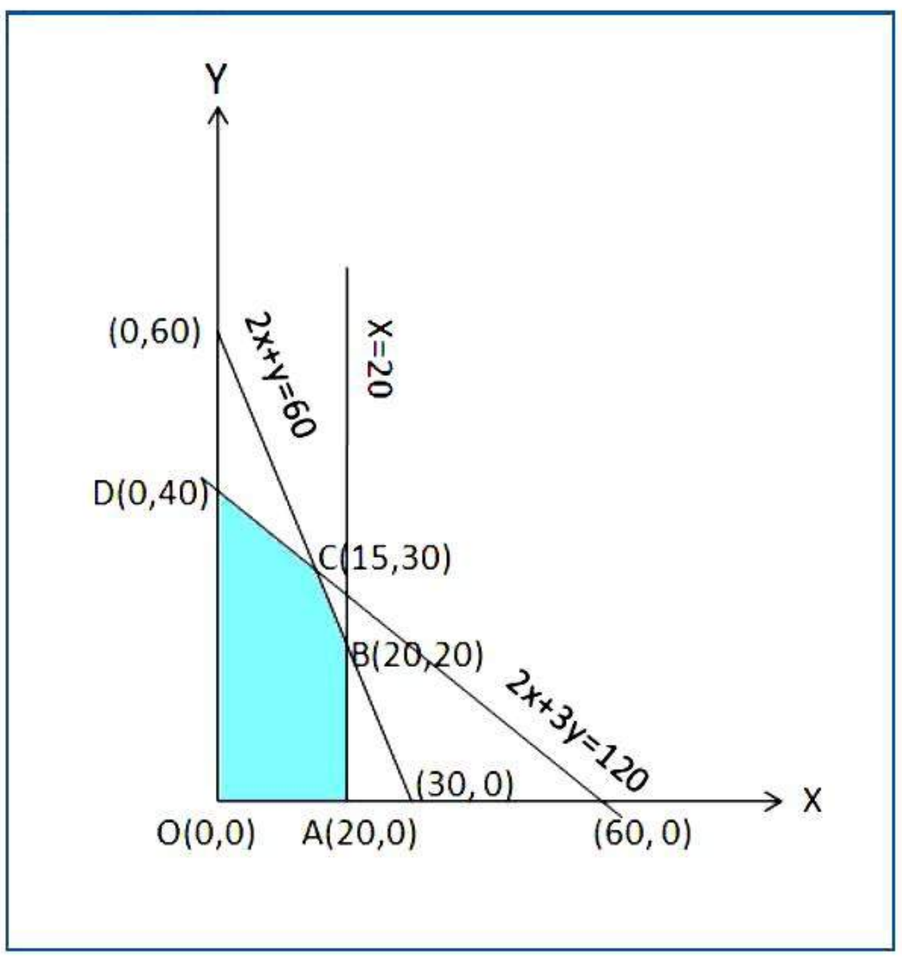
$12x + 6y \leq 360,\ i.e., 2x + y \leq 60.$

$18x + 0\,y \leq 360,\ i.e., x \leq 20$

$6x + 9y \leq 360,\ i.e., 2x + 3y \leq 120$

$x \geq 0,\ y \geq 0$

The feasible region is the polygon OABCD.



The values (in ₹) of $z = 1{\cdot}5x + y$ at the corner points are :

at O(0,0), $z = 0.$

at $A(20,0), z = 1.5 \times 20 + 0 = 30$

at $B(20,20), z = 1{\cdot}5 \times 20 + 1 \times 20 = 50$

at $C(15,30), z = 1{\cdot}5 \times 1.5 + 30 = 52{\cdot}5$

at $D(0,40),\ z = 40$

∴ Maximum profit = ₹ 52·5 when

15 cups of type A and 30 cups of type B are manufactured.

5. The tailors P and Q earn ₹ 150 and ₹ 200 per day respectively. P can stitch 6 shirts and 4 trousers a day, while Q can stitch 10 shirts and 4 trousers per day. How many days should each work to produce at least 60 shirts and 30 trousers at minimum labour cost? (ISC 2012)

Solution : Let the tailor P work for x days and tailor Q work for y days.

We prepare a table from the given data.

Tailor	No of days	No of shirts	No of trousers	Earning (₹)
P	x	6	4	150
Q	y	10	4	200
Requirements		60	32	

The mathematical formulation of the given LPP is

Minimize total labour cost (in ₹) $z = 150x + 200y$

Subject to the constraints

$$6x + 10y \geq 60,\ i.e., 3x + 5y \geq 30$$
$$4x + 4y \geq 32,\ i.e., x + y \geq 8$$
$$x \geq 0$$
$$y \geq 0$$

The diagrammatic representation of the problem is given in the figure below.

The shaded unbounded region ABC is the feasible region.

The corner points are A(10, 0), B(5, 3), C(0, 8) .

The value of $z = 150x + 200y$

at A(10, 0) , $z = 150 \times 10 = 1500$

at B(5, 3), $z = 150 \times 5 + 200 \times 3$
$= 1350$

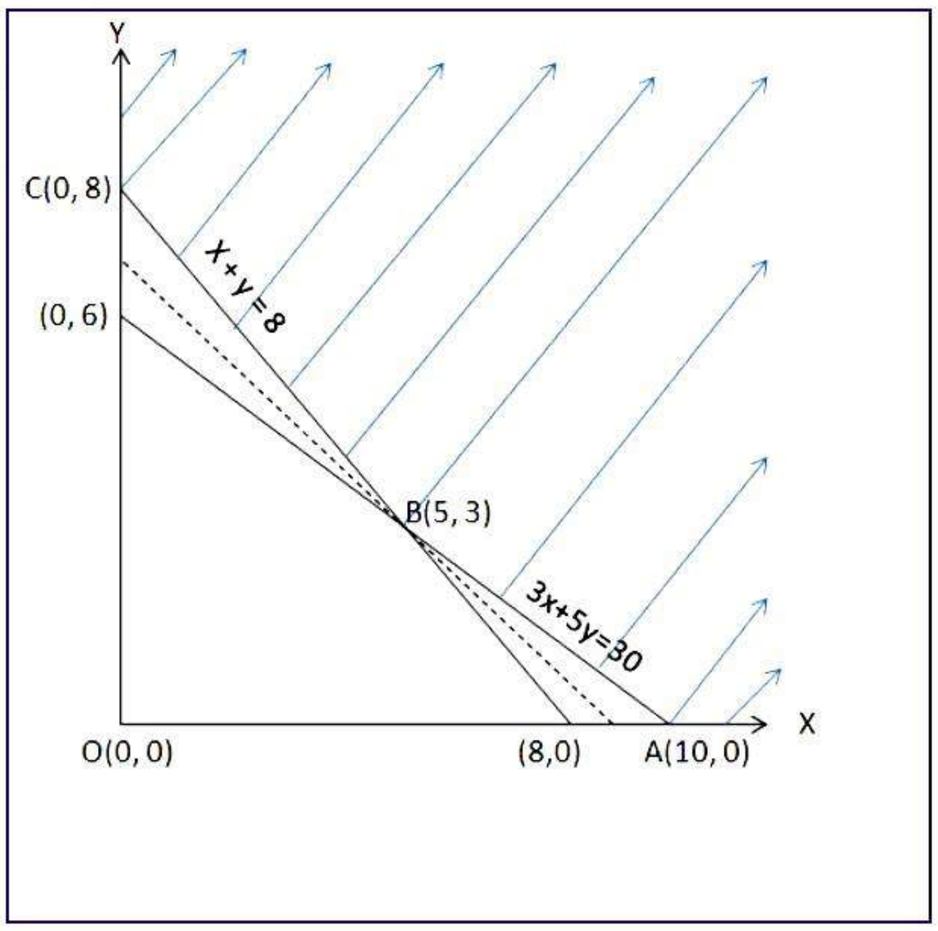
at C(0, 8), $z = 200 \times 8 = 1600$.

Among these values of z , the minimum value is 1350.

Let us draw the line:

$150x + 200y = 1350$. (shown dotted).

The half plane $150x + 200y < 1350$ has no common point with the feasible region. Therefore z has minimum value. Minimum value of z is 1350 and it occurs at the point (5,3).



Hence , the labour cost is minimum when P works for 5 days and Q works for 3 days and the minimum labour cost is ₹ 1350.

6. A mill owner buys two types of machines A and B for his mill. Machine A occupies 1000 sq. m of area and requires 12 men to operate; while machine B occupies 1200 sq. m of area and requires 8 men to operate it. The owner has 7600 sq. m of area available and 72 men to operate the machines. If machine A produces 50 units and machine B produces 40 units daily, how many machines of each

type should he buy to maximize the daily output? Use linear programming to find the solution.
(ISC 2013)

Solution : Let x and y be the number of machines of type A and B respectively which the mill owner should buy. The daily production $P = 50x + 40\,y$ (units).

The information may be stated as follows.

Machines	No of machines	Area (sq. m)	No of men	No of units
A	x	1000	12	50
B	y	1200	8	40
Availability		7600	72	

The problem can be formulated as an LPP as follows:

Maximize $P = 50x + 40y$

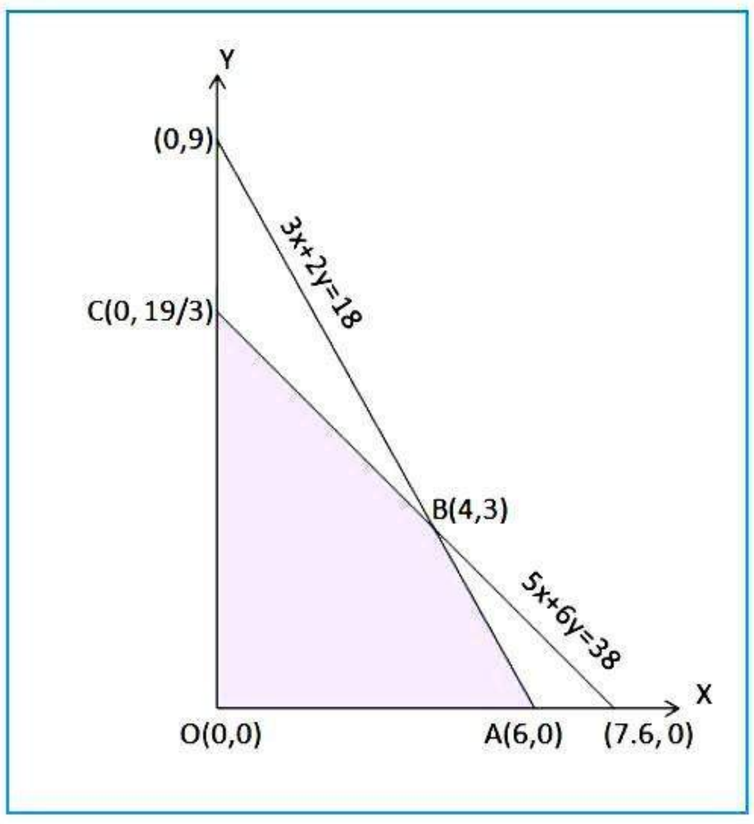
Subject to the constraints

$1000x + 1200y \leq 7600 \quad i.e., \quad 5x + 6y \leq 38$

$12x + 8y \leq 72 \quad i.e., \quad 3x + 2y \leq 18$

$x \geq 0, y \geq 0$.

Shaded portion represents the feasible region.



The corner points are :

$O\ (0,0)\ ,\ A(6,0), B(4,3), C\left(0, \frac{19}{3}\right)$

At $O(0,0), P = 0,$

at $A(6,0), P = 50 \times 6 = 300,$

at $(4,3), P = 50 \times 4 + 40 \times 3 = 320\ ,$

at $C\left(0, \frac{19}{3}\right), P = 40 \times \frac{19}{3} = \frac{760}{3} = 253\frac{1}{3}.$

Hence maximum production P is 320 units at B(4, 3) .

Hence the mill owner should buy 4 machines of type A and 3 machines of type B.

7. A company manufactures two types of toys A and B. A toy of type A requires 5 minutes for cutting and 10 minutes for assembling. A toy of type B requires 8 minutes for cutting and 8 minutes for assembling. There are 3 hours available for cutting and 4 hours available for assembling the toys in a day. The profit is ₹ 50 each on a toy of type A and ₹ 60 each on a toy of type B. How many toys of each type should the company manufacture in a day to maximize the profit? Use linear programming to find the solution. (ISC 2014)

Solution : Let the number of toys of type A be x and the number of toys of type B be y.

The information is given below in tabular form.

Type of toys	Number of toys	Time for cutting	Time for assembling	Profit (₹)
A	x	5 minutes	10 minutes	50
B	y	8 minutes	8 minutes	60
Time available (minutes)		180 minutes	240 minutes	

The problem can be formulated as an LPP as follows.

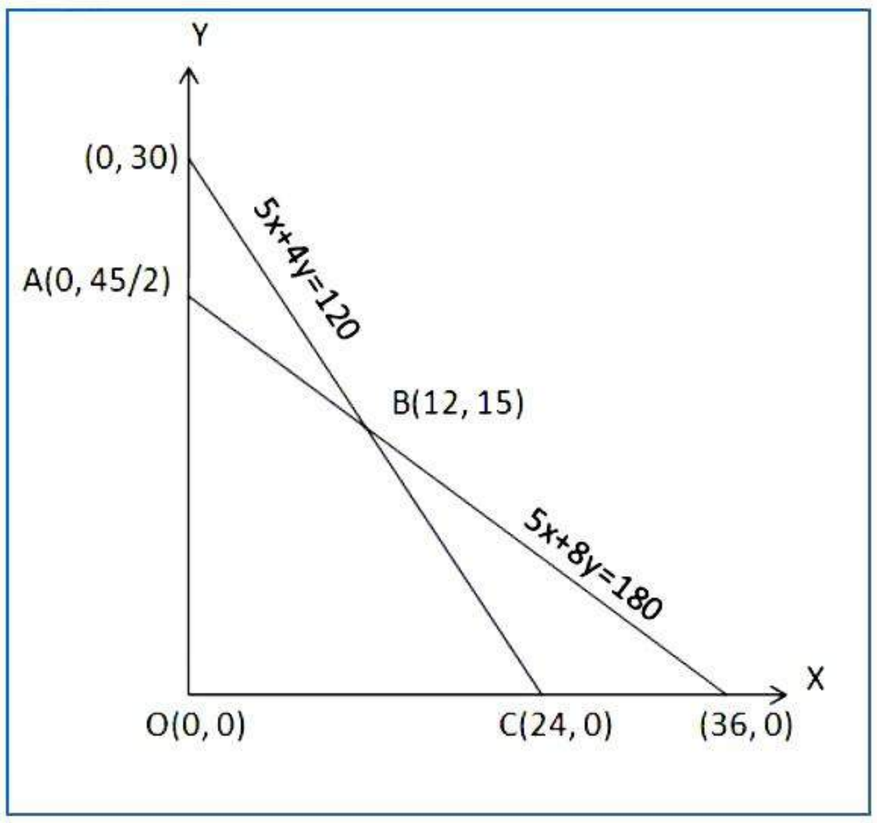
Maximize profit (in ₹) $z = 50x + 60y$

Subject to $5x + 8y \leq 180$

$10x + 8y \leq 240$ $i.e.$ $5x + 4y \leq 120$

$x \geq 0,\ y \geq 0.$

The feasible region is the polygon OABC.



The corner points are :

O(0,0), A(0, 45/2), B(12,15), C(24,0).

At O(0,0), $z = 0$,

at A(0, 45/2), $z = 60 \times \frac{45}{2} = 1350$

at B(12,15), $z = 50 \times 12 + 50 \times 15 = 1500$,

at C(24,0), $z = 50 \times 24 = 1200$.

Hence maximum profit is ₹ 1500 and is obtained when 12 units of type A and 15 units of type B is produced.

8. A dietician wishes to mix two kinds of food X and Y in such a way that the mixture contains at least 10 units of element A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1 unit	2 units	3 units
Y	2 units	2 units	1 unit

One kg of food costs ₹ 24 and one kg of food Y costs ₹ 36. Using linear programming, find the least cost of the total mixture which will contain the required vitamins. (ISC 2015)

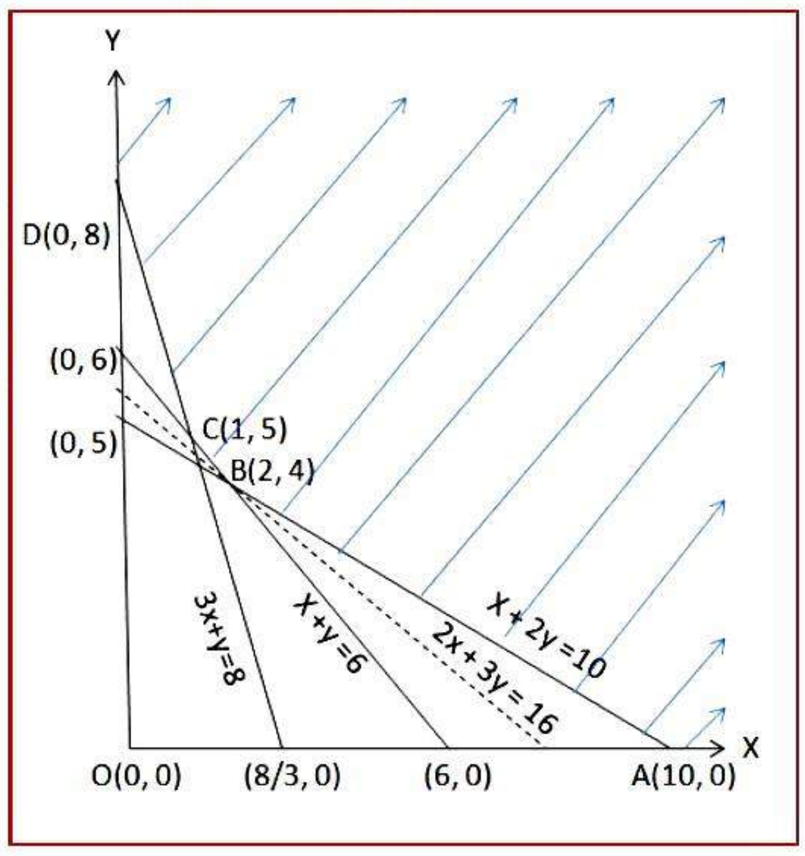
Solution : Let x kg of food X and y kg of food Y be mixed and Z (in ₹) be the total cost of the (mixture) food. Hence , the problem can be formulated as an LPP as follows :

Minimize $Z = 24x + 36y$

Subject to the constraints

$$x + 2y \geq 10\,,$$
$$2x + 2y \geq 12,\ \ i.e., x + y \geq 6$$
$$3x + y \geq 8$$
$$x \geq 0,\ y \geq 0$$

The feasible unbounded region is shown in the figure.



The corner points are A(10,0), B(2,4), C(1,5), D(0,8).

At A(10, 0), $Z = 24 \times 10 = 240$,

at B(2,4), $Z = 24 \times 2 + 36 \times 4 = 192$,

at C(1,5), $Z = 24 \times 1 + 36 \times 5 = 204$,

at D(0,8), $Z = 36 \times 8 = 288$.

Among this value of Z, the minimum value is 192.

We draw the line, $24x + 36y = 192$ i.e, $2x + 3y = 16$. (shown dotted in the figure)

We see that the half plane $2x + 3y < 16$ has no common point with the feasible region.

Hence, Z has minimum value and minimum value of Z is ₹ 192 and it occurs at B(2,4).

Hence, least cost of the mixture is ₹ 192 when 2 kg of food X is mixed with 4 kg of food Y.

9. A company manufactures two types of product A and B. Each unit of A requires 3 grams of nickel and 1 gram of chromium, while each unit of B requires 1 gram of nickel and 2 grams of chromium. The firm can produce 9 grams of nickel and 8 grams of chromium. The profit is ₹ 40 on each unit of product A and ₹ 50 on each unit of type B. How many unit of each type should the company manufacture so as to earn maximum profit? Use linear programming to find the solution. (ISC 2016)

Solution : Let x and y be the number of products of types A and B respectively manufactured by the company. Let Z be the total profit in ₹.

The information given in the problem may be stated as follows:

Product	No of products produced	Materials used per unit (in gms)		Profit in ₹.
		Nickel	Chromium	
A	x	3	1	40
B	y	1	2	50
Materials available		9	8	

The mathematical formulation of the given LPP is given below:

Maximize (in ₹) $Z = 40x + 50y$

subject to the constraints

$$3x + y \leq 9,$$
$$x + 2y \leq 8,$$
$$x \geq 0, y \geq 0.$$

OABC is the shaded bounded feasible region.

The corner points are O(0, 0), A(3, 0), B(2, 3) and C(0, 4).

At O(0, 0), $Z = 0$,

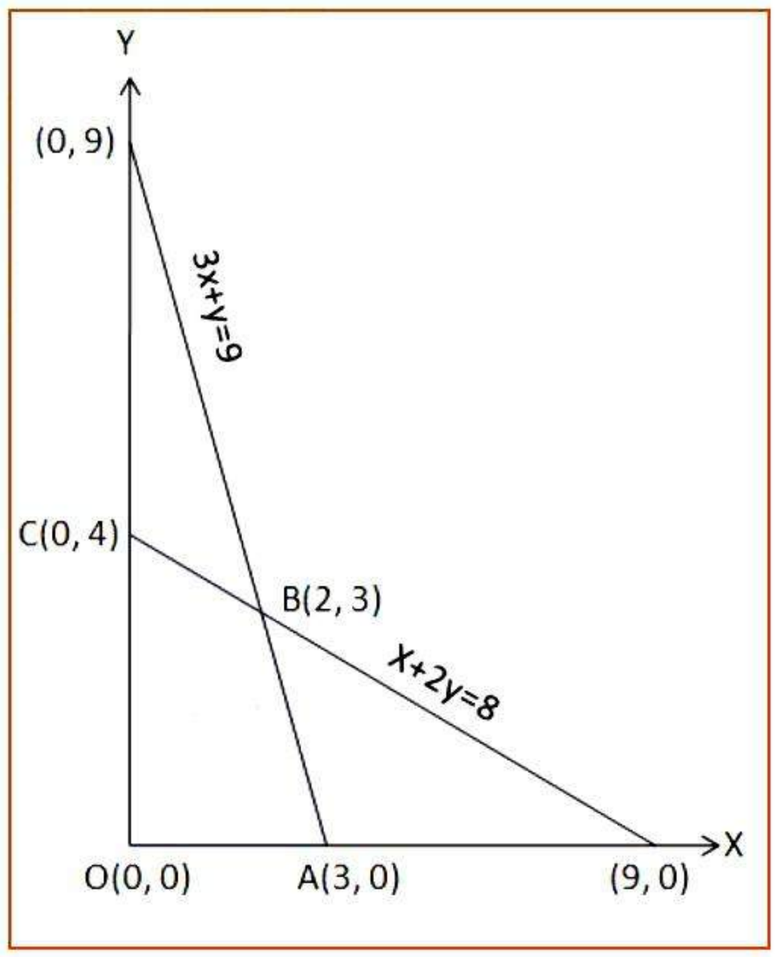
at A(3, 0), $Z = 40 \times 3 = 120$,

at B(2, 3), $Z = 40 \times 2 + 50 \times 3 = 230$,

at C(0, 4), $Z = 50 \times 4 = 200$.

Hence the maximum value of Z is ₹ 230 which is attained at B(2, 3).

Hence , the company should manufacture 2 products of type A and 3 products of type B to get a maximum profit of ₹ 230.



10. A farmer has a supply of chemical fertilizer of type A which contains 10% nitrogen and 6% phosphoric acid and of type B which contains 5% nitrogen 10% phosphoric acid. After soil test , it is found that at least 7 kg of nitrogen and same quantity of phosphoric acid is required for a good crop. The fertilizer of type A costs ₹ 5 per kg and the type B cost ₹ 8 per kg. Using linear programming, find how many kilograms of each type of the fertilizer should be bought to meet the requirement and for the cost to be minimum. Find the feasible region in the graph.

(ISC 2017)

Solution : Let the fertilizers of type A and type B be x kg and y kg respectively.

The given information is tabulated as follows:

Fertilizer	Amount in kg	Nitrogen	Phosphoric acid	Costs in ₹
A	x	10%	6%	5
B	y	5%	10%	8
Minimum requirement		7 kg	7 kg	

Mathematical formulation of the given LPP is given below:

Minimize cost (in ₹) $C = 5x + 8y$

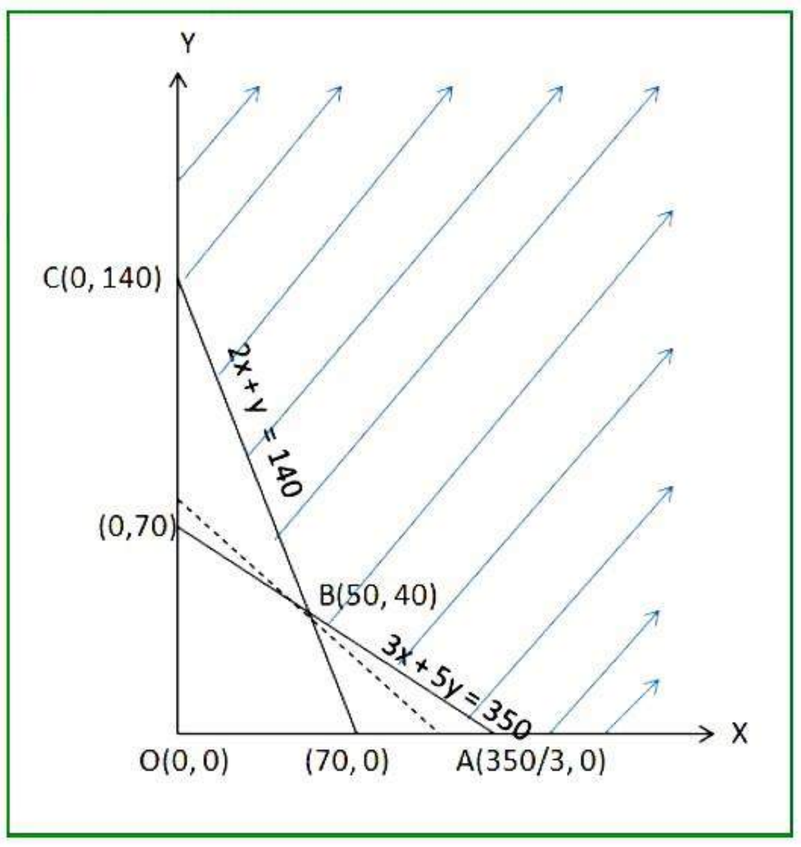
subject to the constraints

$$\frac{10}{100}x + \frac{5}{100}y \geq 7 \quad i.e.\ 2x + y \geq 140\ .$$

$$\frac{6}{100}x + \frac{10}{100}y \geq 7 \quad i.e.\ 3x + 5y \geq 350$$

$$x \geq 0\ ,\ y \geq 0.$$

The feasible unbounded region is shown in the figure.



The corner points are A(350/3, 0), B(50, 40), C(0, 140).

At $A\left(\frac{350}{3}, 0\right), C = 5 \times \frac{350}{3} = \frac{1750}{3} = 583\frac{1}{3}$

At $B(50,40), C = 5 \times 50 + 8 \times 40 = 570$

At $C(0, 140), C = 8 \times 140 = 1100$

Among these value of C, minimum value of C is ₹ 570.

Let us draw the line $5x + 8y = 570$. (shown dotted).

The half plane $5x + 8y < 570$ has no common point with the feasible region. Therefore, C has minimum value.

The minimum value of C is ₹ 570 and it occurs at (50, 40).

Hence, the farmer should bought 50 kg of type A fertilizer and 40 kg of type B fertilizer so that the cost is minimum and minimum cost is ₹ 350.

11. A manufacturing company makes two types of teaching aids A and B of Mathematics for class X. Each type of A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹ 80 on each piece of type of A and ₹ 120 on each piece of type B. How many

pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically. What is the maximum profit per week? (ISC 2018)

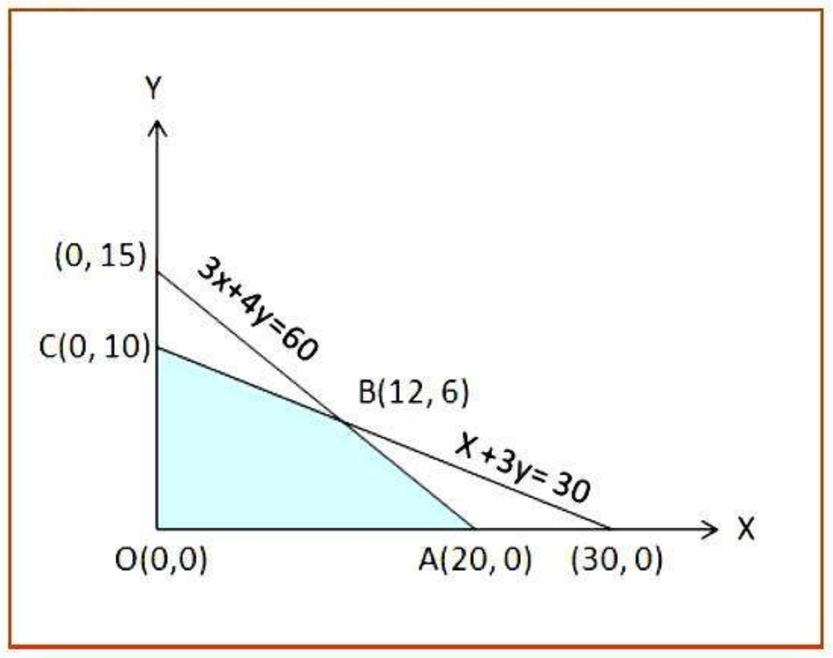
Solution : Let x and y be the number of teaching aids of type A and B respectively.
The given information may be tabulated as below:

Teaching aids	Number of pieces	Time in hours		Profit (₹)
		Fabricating	finishing	
A	x	9	1	80
B	y	12	3	120
Available time (in hours)		180	30	

The problem can be formulated as an LPP as follows:
Maximize Profit (in ₹) $P = 80\,x + 120\,y$
Subject to the constraints

$$9x + 12\,y \leq 180 \;\; i.e. 3x + 4y \leq 60$$
$$x + 3y \leq 30$$
$$x \geq 0,\; y \geq 0\,.$$

The shaded region shows the feasible region which is bounded.



The corner points of the feasible region OABC are O(0, 0), A(20, 0), B(12, 6), C(0, 10).

At O(0, 0), $P = 0$
At A(20, 0), $P = 80 \times 20 = 1600$
At B(12, 6), $P = 80 \times 12 + 120 \times 6 = 1680$
At C(0, 10), $P = 120 \times 10 = 1200$
Hence value of P is maximum at B(12, 6) .
Hence, the manufacturer should produce 12 teaching aids of type A and 6 teaching aids of type B to get a maximum of profit ₹ 1680.

12. A carpenter has 90, 80 and 50 running feet respectively of teak wood, plywood and rosewood which is used to produce product A and product B. Each unit of product A requires 2, 1 and 1 running feet and each unit of product B requires 1, 2 and 1 running feet of teak wood, plywood and rosewood respectively. If product A is sold for ₹ 48 per unit and product B is sold for ₹ 40 per unit, how many

units of product A and product B should be produced and sold by the carpenter , in order to obtain the maximum gross income?

Formulate the above as a Linear Programming Problem and solve it, indicating clearly the feasible region in the graph. (ISC 2019)

Solution : Let the number of units of the product A to be made be x and that of the product B to be made be y. Let z be the gross income in ₹. The information may be given as a tabular form as follows:

Product	No of units to be made	Wood used per unit (ft.)			Income per unit (₹)
		Teak	Plywood	rosewood	
A	x	2	1	1	48
B	y	1	2	1	40
Available stock		90	80	50	

Mathematical formulation of the LPP :

Maximize (in ₹) $z = 48\,x + 40\,y$

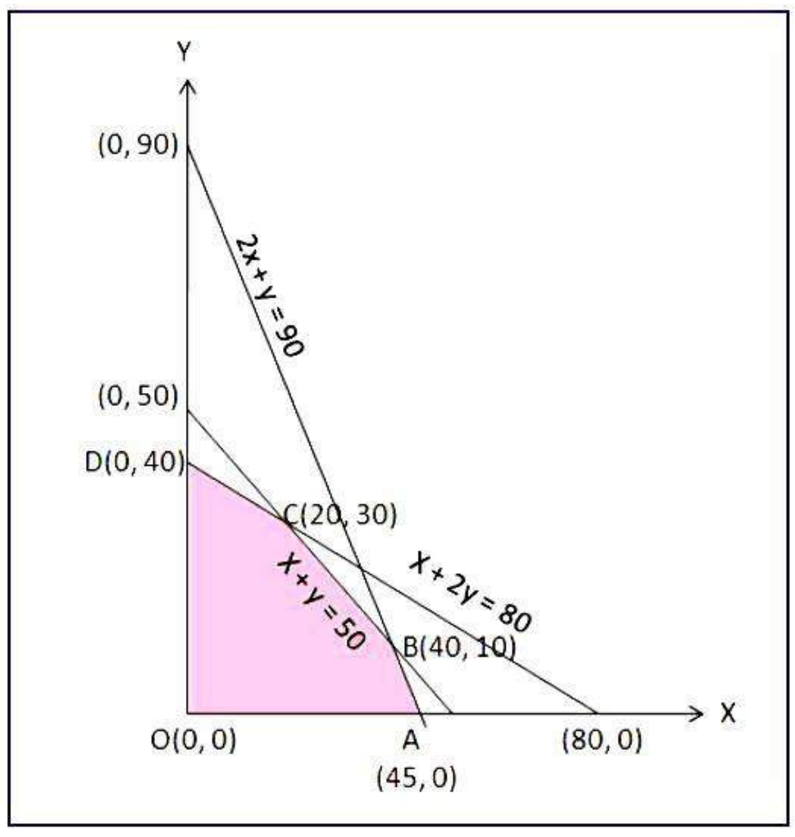
Subject to the constraints :

$$2x + y \leq 90$$

$$x + 2y \leq 80$$

$$x + y \leq 50$$

$$x \geq 0, y \geq 0.$$



The shaded convex polygon OABCD is the feasible region.

The corner points are :

O(0, 0), A(45, 0), B(40, 10), C(20, 30), D(0, 40).

At O(0, 0), $z = 0$.

At A(45, 0), $z = 48 \times 45 = 2160$

At B(40, 10), $z = 48 \times 40 + 40 \times 10 = 2320$

At C(20, 30), $z = 48 \times 20 + 40 \times 30 = 2160$

At D(0, 40), $z = 40 \times 40 = 1600$.

Hence maximum value of z is ₹ 2320 and it is obtained at B(40, 10). Hence for maximum gross income , the carpenter should make 40 units of product A and 10 units of product B.

13. A company uses three machines to manufacture two types of shirts, half sleeves and full sleeves. The number of hours required per week on machine M_1 , M_2 and M_3 for one shirt of each type is given in the following table :

	M_1	M_2	M_3
Half sleeves	1	2	8/5
Full sleeves	2	1	8/5

None of the machines can be in operation for more than 40 hours per week. The profit on each half sleeve shirt is ₹ 1 and the profit on each full sleeve shirt is ₹ $1{\cdot}50$. How many of each type of shirts should be made per week to maximixe the company's profit?

Solution : Let x and y be the number of half sleeves and full sleeves shirts manufactured by the company per week respectively to maximize the profit.

Mathematical formulation of the LPP :

Maximize profit (in ₹) $z = x + 1{\cdot}50\, y$

subject to the constraints

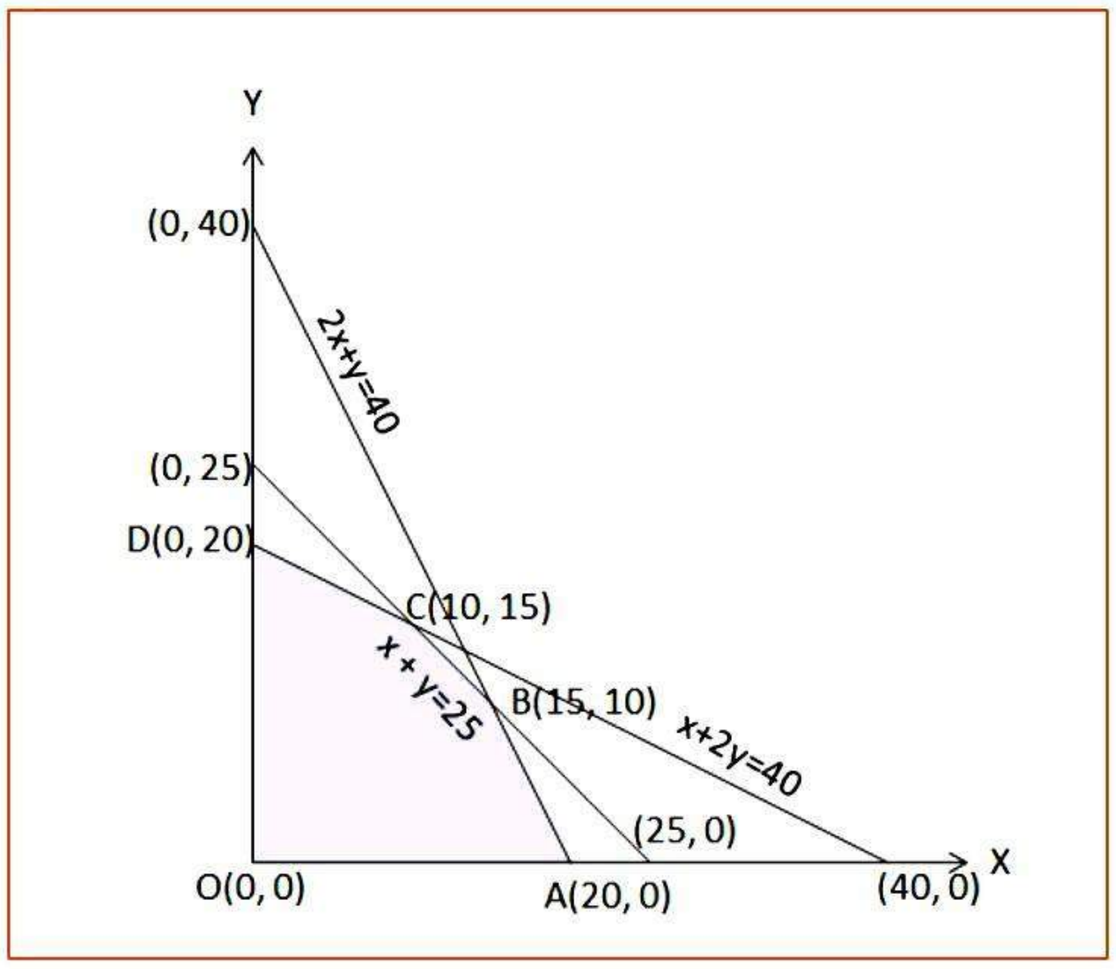
$x + 2y \leq 40$

$2x + y \leq 40$

$\frac{8}{5}x + \frac{8}{5}y \leq 40 \quad i.e.,\ x + y \leq 25$

$x \geq 0$ and $y \geq 0$.

The feasible region is the polygon OABCD which is bounded.



The corner points are O(0, 0), A(20, 0), B(15, 10), C(10, 15) and D(0, 20).

The values (in ₹) of $z = x + 1{\cdot}5\, y$ at corner points are :

At O(0, 0), $z = 0$

At A(20, 0), $z = 20$

At B(15, 10), $z = 15 + 1{\cdot}5 \times 10 = 30$

At C(10, 15), $z = 10 + 1{\cdot}5 \times 15 = 10 + 22.5 = 32{\cdot}5$

At D(0, 20), $z = 1{\cdot}5 \times 20 = 30$

∴ Maximum profit = ₹ $32{\cdot}5$ and is obtained at C(10, 15) i.e., when 10 half sleeves shirts and 15 full sleeves shirts are made by the company per week.

14. A Manufacturer has two machines X and Y that may run at the most 360 minutes in a day to produce two types of toys A and B.
To produce each Toy A, machines X and Y need to run at the most 12 minutes and 6 minutes respectively.
To produce each toy B, machines X and Y need to run at the most 6 minutes and 9 minutes respectively.
By selling the toys A and B, the manufacturer makes the profits of ₹30 and ₹20 respectively.
Formulate a Linear Programming Problem and find the number of toys A and B that should be manufactured in a day to get maximum profit. (ISC Sem 2-2022)

Solution: Let the manufacturer manufactured x number of toys of type A and y number of toys of type B per day. Let z denote total profit in rupees.

Type of toys	Number of toys	Manufacturing time (minutes)		Profit in ₹
		Machine X	Machine Y	
A	x	12	6	30
B	y	6	9	20
Time available		360	360	

The mathematical formulation of the given LPP is:

Maximize Profit (in ₹), $z = 30\,x + 20\,y$

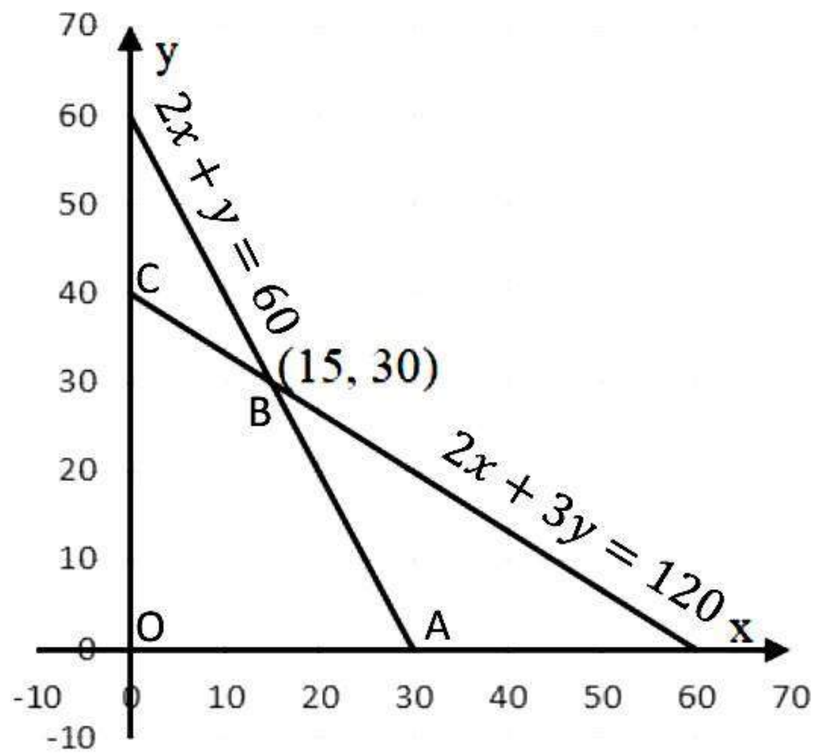
Subject to the constraints

$12x + 6y \leq 360$ or $2x + y \leq 60$

$6x + 9y \leq 360$ or $2x + 3y \leq 120$

$x \geq 0, \quad y \geq 0$

OABC is the required feasible region.



Corner points are O (0,0),
A (30, 0), B (15, 30) and C (0,40).

At O (0, 0), $z = 0$
At A (30, 0), $z = 900$
At B (15, 30), $z = 30 \times 15 + 20 \times 30 = 1050$
At C (0, 40), $z = 20 \times 40 = 800$
Hence at point B (15, 30), z is maximum and maximum profit is ₹ 1050.

Hence when 15 toys of type A and 30 toys of type B are manufactured per day, profit is maximum.